Improved Measurement of $\mathcal{B}(D^+ \to \mu^+\nu)$ and the Pseudoscalar Decay Constant f_{D^+}

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Abstract

We extract a relatively precise value for the decay constant of the D^+ meson by measuring $\mathcal{B}(D^+ \to \mu^+\nu) = (4.40 \pm 0.66^{+0.09}_{-0.12}) \times 10^{-4}$ using 281 pb^{-1} of data taken on the $\psi(3770)$ resonance with the CLEO-c detector. We find $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV, and compare with current theoretical calculations. We also set a 90% confidence upper limit on $\mathcal{B}(D^+ \to e^+\nu) < 2.4 \times 10^{-5}$ which constrains new physics models.

Purely leptonic pseudoscalar meson decays proceed via the annihilation of the constituent quarks into a virtual W via the axial-vector current. The decay rate is proportional to the square of the decay constant f, a single number which encapsulates strong interaction dynamics in the decay. The lack of hadrons in the final state allows for precision tests of strong interaction theories. Knowledge of decay constants is critical for extracting fundamental information about CKM matrix elements. For example, f_B is needed to use measurements of $B\bar{B}$ mixing. Currently, it is not possible to determine f_B experimentally, so theoretical calculations must be used. The most promising of these calculations involves lattice QCD [1], though there are other methods. Measurements of pseudoscalar decay constants from charm meson decays provide checks on these calculations and can discriminate among different models.

The decay $D^+ \to \ell^+ \nu$ proceeds by the c and $\bar{d}$ quarks annihilating into a virtual W^+, with a decay width [2]

$$\Gamma(D^+ \to \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D^+}^2 m_\ell^2 M_{D^+} \left(1 - \frac{m_\ell^2}{M_{D^+}^2}\right)^2 |V_{cd}|^2 \quad , \tag{1}$$

where M_{D^+} is the D^+ mass, m_ℓ is the mass of the final state lepton, $|V_{cd}|$ is a CKM matrix element that we assume to be equal to $|V_{us}|$, and G_F is the Fermi coupling constant. Because of helicity suppression, the rate is a function of m_ℓ^2; consequently, the electron mode $D^+ \to e^+ \nu$ has a very small rate in the standard model. The relative widths are $2.65 : 1 : 2.3 \times 10^{-5}$ for the $\tau^+ \nu$, $\mu^+ \nu$, and $e^+ \nu$ final states, respectively.

The CLEO-c detector is equipped to measure the momenta and directions of charged particles, identify them using specific ionization (dE/dx) and Cherenkov light (RICH), detect photons and determine their directions and energies [3].

In this study we use 281 pb^{-1} of data produced in e^+e^- collisions using the Cornell Electron Storage Ring (CESR) and recorded at the ψ'' resonance (3.770 GeV). This work contains our previous sample as a subset and supersedes our initial effort [4]. At this energy, the events consist mostly of D^+D^-, $D^0\overline{D}^0$, three-flavor continuum $q\bar{q}$ ($q = u, d, s$), $\tau^+\tau^-$, and $\gamma\psi'$ events.

Our analysis strategy is to fully reconstruct the D^- meson in one of six decay modes listed in Table I and search for a $D^+ \to \mu^+\nu$ decay in the rest of the event. Charge conjugate modes are implicitly included throughout the paper. Track selection, particle identification (PID), π^0, K_S, and muon selection cuts are identical to those used in Ref. [4]. We first evaluate ΔE, the difference in the energy of the decay products with the beam energy. The ΔE distributions in all modes are well described by either a Gaussian or the sum of two Gaussians, with root mean square (r.m.s.) widths varying from 7 MeV for $D^- \to K^+K^-\pi^-$ to 14 MeV for $D^- \to K^+\pi^-\pi^-\pi^0$. We select candidates by requiring $|\Delta E| < 0.012 - 0.024$ GeV, where the cut in each mode is approximately 2.5 times the r.m.s. width.

For the selected events we construct the beam-constrained mass, $m_{BC} = \sqrt{E_{\text{beam}}^2 - (\sum_i \boldsymbol{p}_i)^2}$, where i runs over the final state particles from the candidate D^- decay. The resolution in m_{BC} of 2.2-2.4 MeV is better than merely calculating the invariant mass of the decay products, since the CESR beam has a small energy spread. The m_{BC} distribution for the sum of all D^- tagging modes is shown in Fig. 1. The numbers of tags in each mode are determined from fits of the m_{BC} distributions to a signal function plus a background shape. For the latter we use an expression analogous to one first used by the

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ARGUS collaboration to approximate the correct threshold behavior [5]. For the signal we use an asymmetric lineshape because of the tail towards high mass caused by initial state radiation [6]. Table I gives the numbers of signal and background events for each mode within the signal region, defined as $m_D - 2.5\,\sigma_{m_{BC}} < m_{BC} < m_D + 2.0\,\sigma_{m_{BC}}$, where $\sigma_{m_{BC}}$ is the r.m.s. width of the lower side of the distribution.

Mode	Signal	Background
$K^+\pi^-\pi^-$	77387 ± 281	1868
$K^+\pi^-\pi^-\pi^0$	24850 ± 214	12825
$K_S\pi^-$	11162 ± 136	514
$K_S\pi^-\pi^-\pi^+$	18176 ± 255	8976
$K_S\pi^-\pi^0$	20244 ± 170	5223
$K^+K^-\pi^-$	6535 ± 95	1271
Sum	158354 ± 496	30677

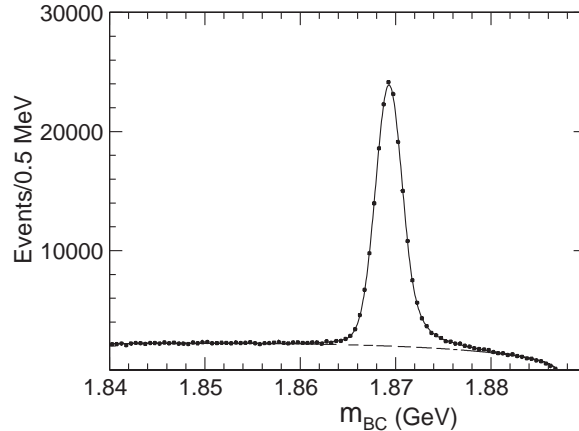TABLE I: Tagging modes and numbers of signal and background events.

FIG. 1: Beam-constrained mass for the sum of fully reconstructed D^- decay candidates. The solid curve shows the fit to the sum of signal and background functions, while the dashed curve indicates the background.

Using our sample of D^- candidates we search for events with a single additional charged track presumed to be a μ^+. The track must make an angle $>35.9°$ with respect to the beamline, deposit less than 300 MeV of energy in the calorimeter, characteristic of a minimum ionizing particle, and not be identified as a kaon. We then calculate

$$\mathrm{MM}^2 = (E_{\mathrm{beam}} - E_{\mu^+})^2 - \left(-\boldsymbol{p}_{D^-} - \boldsymbol{p}_{\mu^+}\right)^2, \tag{2}$$

where $\boldsymbol{p}_{D^-}$ is the three-momentum of the fully reconstructed D^-. Real $D^+ \rightarrow \mu^+\nu$ events will congregate near zero MM^2.

The MM^2 from Monte Carlo simulation has a resolution (two-Gaussian σ) of 0.0235 ± 0.0004 GeV^2 consistent among all the tagging decay modes. We check our simulations by using the $D^+ \rightarrow K_S\pi^+$ decay. Here we choose events with the same requirements

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as used to search for $\mu^+\nu$ but require one additional found K_S. The resolution is measured to be 0.0233 ± 0.0009 GeV^2, consistent with the Monte Carlo estimate.

In order to restrict the sample to candidate $\mu^+\nu$ events, we impose restrictions on tracks and neutral energy clusters in addition to those used to reconstruct the tagging D^-. We select events with only one additional charged track; events with extra tracks originating within 0.5 m (radially) of the event vertex are rejected. In addition, we eliminate events having at least one extra neutral energy cluster of more than 250 MeV. These cuts are highly effective in reducing backgrounds especially from $D^+ \to \pi^+\pi^0$ decays, but they introduce an inefficiency because the decay products of the tagging D^- can interact in the detector material leaving spurious tracks or clusters. To evaluate our cut efficiencies, we use an essentially background-free sample of fully reconstructed D^+D^- events. (The method is different here than in our original publication, though the results are consistent.)

To first order, the fully reconstructed $D^-D^+ \to K^+\pi^-\pi^-$, $K^-\pi^+\pi^+$ events can be considered the superposition of two $D^-D^+ \to K^+\pi^-\pi^-$, $\mu^+\nu$ events. Our procedure is to evaluate the cut efficiency in our sample of 1435 events and take the square-root. This gives us the efficiency for the $D^- \to K^+\pi^-\pi^-$ tag sample. We then combine the $K^+\pi^-\pi^-$ with each of the other tags in turn. This method ensures that the number of interactions of particles with material and their resulting effects is the same as in the tag sample used for the $\mu^+\nu$ analysis. In the sample of fully reconstructed D^-D^+ events there are no events with extra tracks originating within 0.5 m of the main event vertex. The efficiency for rejecting events with extra clusters above 250 MeV, averaging over all our tag modes, is $(96.1\pm0.3\pm0.4)\%$ The systematic error arises only because we have analyzed a situation corresponding to two overlapping tags rather than one tag plus a muon. In Monte-Carlo simulation the efficiency difference is 0.4%, which we assign as a systematic error.

The MM^2 distribution is shown in Fig. 2. We see a peak near zero containing 50 events within the interval -0.050 GeV^2 to $+0.050$ GeV^2, approximately $\pm2\sigma$ wide. The peak is mostly due to $D^+ \to \mu^+\nu$ signal. The large peak centered near 0.25 GeV^2 is from the decay $D^+ \to \overline{K}^0\pi^+$ that is far from our signal region and is expected, since many K_L escape our detector.

There are several potential background sources; these include other D^+ modes, misidentified $D^0\overline{D}^0$ events, and continuum including $e^+e^- \to \gamma\psi'$. Hadronic sources need to be considered because the requirement of the muon depositing less than 300 MeV in the calorimeter, while about 99% efficient on muons, rejects only about 40% of pions or kaons as determined from a pure sample of $D^0 \to K^-\pi^+$ decays.

There are a few specific D^+ decay modes that contribute unwanted events in the signal region. Residual $\pi^+\pi^0$ background is determined from a simulation that uses a branching fraction of $(0.13\pm0.02)\%$ [7] and yields $1.40\pm0.18\pm0.22$ events; the first error is due to Monte Carlo statistics, and the second is systematic, due mostly to the branching ratio uncertainty. We find background from $D^+ \to \tau^+\nu$ only when $\tau^+ \to \pi^+\nu$. Since the $\tau^+\nu$ branching ratio is known to be 2.65 times the $\mu^+\nu$ rate from Eq. 1, our simulation gives $1.08\pm0.15\pm0.16$ events, where the systematic error arises from our final uncertainty on the $\mu^+\nu$ decay rate. The $\overline{K}^o\pi^+$ mode (branching ratio of $(2.77\pm0.18)\%$ [8]) gives a large peak in the MM^2 spectrum near 0.25 GeV^2. While far from our signal region, the tail of the distribution can contribute. We measure this background rate directly using $D^0\overline{D}^0$ events. Here we select a sample of single tags, either $K^-\pi^+\pi^+\pi^-$, $K^-\pi^+\pi^0$ or $K^-\pi^+$ and look for events with only two additional oppositely signed tracks, one identified as a kaon and one as a pion using the RICH. We
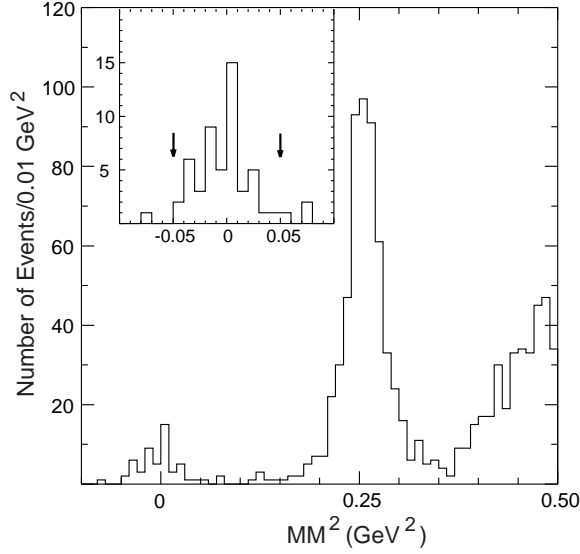
FIG. 2: MM^2 using D^- tags and one additional opposite sign charged track and no extra energetic clusters (see text). The insert shows the signal region for $D^+ \to \mu^+\nu$ enlarged; the defined signal region is shown between the two arrows.

then compute the MM^2 ignoring the kaon. The MM^2 distribution shows a narrow peak near 0.25 GeV^2 and three events in the signal region, corresponding to a background of 0.33±0.19±0.02 events, the systematic error being due to the branching ratio uncertainty. (A simulation gives a consistent estimate of 0.44±0.22 events.)

We have also checked the possibility of other D^+D^- decay modes producing background with an equivalent 1.7 fb^{-1} Monte Carlo sample. We evaluate $D^0\overline{D}^0$ and continuum backgrounds by analyzing Monte Carlo samples corresponding to 0.54 fb^{-1}. To normalize our Monte Carlo events to our data sample, we use $\sigma_{D^0\overline{D}^0} = 3.5$ nb and $\sigma_{\text{continuum}} = 14.5$ nb [9]. No additional background events are found in any of these samples.

Our total background is 2.81±0.30±0.27 events. The backgrounds from other D^+, D^0, and continuum sources are limited to less than 0.4, 0.4, and 1.2 events at 90% confidence level (C.L.), respectively. To account for possible backgrounds from these sources, we add them as 32% C.L. (1σ) values in quadrature for a positive error and therefore add an additional $^{+0.8}_{-0}$ event systematic error.

We have $47.2\pm7.1^{+0.3}_{-0.8}$ $\mu^+\nu$ signal events after subtracting background. The detection efficiency for the single muon of 69.4% includes the selection on MM^2 within $\pm2\sigma$ limits, the tracking, the particle identification, probability of the crystal energy being less than 300 MeV, and corrections for final state radiation [10]. It does not include the 96.1% efficiency of not having another unmatched cluster in the event with energy greater than 250 MeV. We also need to account for the fact that it is easier to find tags in $\mu^+\nu$ events than in generic decays by a small amount, (1.5±0.4±0.5)%, as determined by Monte Carlo simulation.

Our result for the branching fraction, using the tag sum in Table I, is

$$\mathcal{B}(D^+ \to \mu^+\nu) = (4.40 \pm 0.66^{+0.09}_{-0.12}) \times 10^{-4} \ . \tag{3}$$

The systematic errors on the branching ratio are listed in Table II. (The systematic error on the tag sum is estimated from varying the signal and background functions.)

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	Systematic errors (%)
MC statistics	±0.4
Track finding	±0.7
PID cut	±1.0
MM^2 width	±1.0
Minimum ionization cut	±1.0
Number of tags	±0.6
Extra showers cut	±0.5
Background	+0.6, −1.7
Total	+2.1, −2.5

TABLE II: Systematic errors on the $D^+ \to \mu^+\nu$ branching ratio.

The decay constant f_{D^+} is then obtained from Eq. (1) using 1.040±0.007 ps as the D^+ lifetime [8], and $|V_{cd}| = 0.2238 \pm 0.0029$ [11]. (We add these two small additional sources of uncertainty into the systematic error.) Our final result is

$$f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4}) \text{ MeV} . \tag{4}$$

We use the same tag sample to search for $D^+ \to e^+\nu_e$. We identify the electron using a match between the momentum measurement in the tracking system and the energy deposited in the CsI calorimeter as well as insuring that the shape of the energy distribution among the crystals is consistent with that expected for an electromagnetic shower. Other cuts remain the same. We do not find any candidates, yielding a 90% C.L. limit of $\mathcal{B}(D^+ \to e^+\nu_e) < 2.4 \times 10^{-5}$, including systematic errors.

Our measurement of f_{D^+} is much more precise than previous observations or limits [4, 12]. The theoretical predictions listed in Table III were made prior to this result. The first entry is the result from the Fermilab-MILC-HPQCD collaboration that is done with all three light quark flavors unquenched, hence n_f=2+1 [13]. It is about 10% smaller than our result, albeit within error.

The models generally predict $f_{D_s^+}$ to be 10–25% larger than f_{D^+} which is consistent with a previous CLEO measurement [22]. Some non-standard models predict significant rates for the helicity suppressed decay $D^+ \to e^+\nu$ [23]. Our upper limit restricts these models.

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Model	f_{D^+} (MeV)	$f_{D_S^+}/f_{D^+}$
Lattice (n_f=2+1) [13]	$201 \pm 3 \pm 17$	$1.24 \pm 0.01 \pm 0.07$
QL (Taiwan) [14]	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [15]	$210 \pm 10^{+17}_{-16}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [16]	$211 \pm 14^{+0}_{-12}$	1.10 ± 0.02
QCD Sum Rules [17]	203 ± 20	1.15 ± 0.04
QCD Sum Rules [18]	195 ± 20	
Quark Model [19]	243 ± 25	1.10
Potential Model [20]	238	1.01
Isospin Splittings [21]	262 ± 29	

TABLE III: Theoretical predictions of f_{D^+} and $f_{D_S^+}/f_{D^+}$. QL indicates quenched lattice calculations.

[5] The function is $f(m_{BC}) = A(m_{BC} + B)\sqrt{1 - \left(\frac{m_{BC}+B}{C}\right)^2} e^{D\left(1 - \left[\frac{m_{BC}+B}{C}\right]^2\right)}$. Here A is the overall normalization and B, C and D are parameters that govern the shape. See H. Albrecht *et al.* (ARGUS), Phys. Lett. B **229**, 304 (1989).

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